Trigonometry

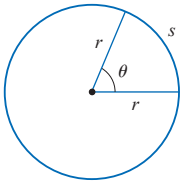
Angle Measurement

π radians $= 180°$

$1° = \dfrac{\pi}{180}$ rad $\qquad 1 \text{ rad} = \dfrac{180°}{\pi}$

$s = r\theta$

(θ in radians)

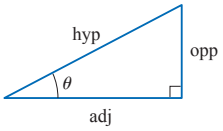


Right Angle Trigonometry

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \dfrac{\text{hyp}}{\text{opp}}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \dfrac{\text{hyp}}{\text{adj}}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} \qquad \cot\theta = \dfrac{\text{adj}}{\text{opp}}$

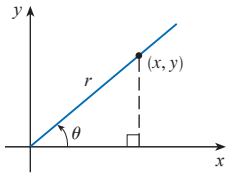


Trigonometric Functions

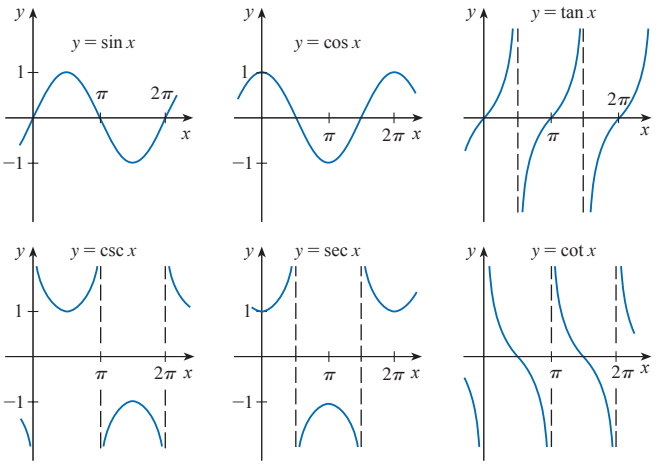
$\sin\theta = \dfrac{y}{r} \qquad \csc\theta = \dfrac{r}{y}$

$\cos\theta = \dfrac{x}{r} \qquad \sec\theta = \dfrac{r}{x}$

$\tan\theta = \dfrac{y}{x} \qquad \cot\theta = \dfrac{x}{y}$



Graphs of Trigonometric Functions



Trigonometric Functions of Important Angles

θ	radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	0	1	0
30°	$\pi/6$	1/2	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\pi/3$	$\sqrt{3}/2$	1/2	$\sqrt{3}$
90°	$\pi/2$	1	0	—

Fundamental Identities

$\csc\theta = \dfrac{1}{\sin\theta} \qquad \sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} \qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\cot\theta = \dfrac{1}{\tan\theta} \qquad \sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$

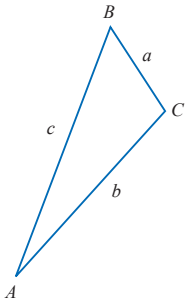
$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta \qquad \sin\left(\dfrac{\pi}{2} - \theta\right) = \cos\theta$

$\cos\left(\dfrac{\pi}{2} - \theta\right) = \sin\theta \qquad \tan\left(\dfrac{\pi}{2} - \theta\right) = \cot\theta$

The Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



The Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Addition and Subtraction Formulas

$\sin(x + y) = \sin x\cos y + \cos x\sin y$

$\sin(x - y) = \sin x\cos y - \cos x\sin y$

$\cos(x + y) = \cos x\cos y - \sin x\sin y$

$\cos(x - y) = \cos x\cos y + \sin x\sin y$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x\tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x\tan y}$$

Double-Angle Formulas

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Half-Angle Formulas

$$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$$

Differentiation Rules

General Formulas

1. $\frac{d}{dx}(c) = 0$

2. $\frac{d}{dx}[cf(x)] = cf'(x)$

3. $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

4. $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

5. $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$ (Product Rule)

6. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ (Quotient Rule)

7. $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ (Chain Rule)

8. $\frac{d}{dx}(x^n) = nx^{n-1}$ (Power Rule)

Exponential and Logarithmic Functions

9. $\frac{d}{dx}(e^x) = e^x$

10. $\frac{d}{dx}(a^x) = a^x \ln a$

11. $\frac{d}{dx}\ln|x| = \frac{1}{x}$

12. $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

Trigonometric Functions

13. $\frac{d}{dx}(\sin x) = \cos x$

14. $\frac{d}{dx}(\cos x) = -\sin x$

15. $\frac{d}{dx}(\tan x) = \sec^2 x$

16. $\frac{d}{dx}(\csc x) = -\csc x \cot x$

17. $\frac{d}{dx}(\sec x) = \sec x \tan x$

18. $\frac{d}{dx}(\cot x) = -\csc^2 x$

Inverse Trigonometric Functions

19. $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1 - x^2}}$

20. $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1 - x^2}}$

21. $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1 + x^2}$

22. $\frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2 - 1}}$

23. $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2 - 1}}$

24. $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1 + x^2}$

Hyperbolic Functions

25. $\frac{d}{dx}(\sinh x) = \cosh x$

26. $\frac{d}{dx}(\cosh x) = \sinh x$

27. $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

28. $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$

29. $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

30. $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

Inverse Hyperbolic Functions

31. $\frac{d}{dx}(\sinh^{-1}x) = \frac{1}{\sqrt{1 + x^2}}$

32. $\frac{d}{dx}(\cosh^{-1}x) = \frac{1}{\sqrt{x^2 - 1}}$

33. $\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1 - x^2}$

34. $\frac{d}{dx}(\operatorname{csch}^{-1}x) = -\frac{1}{|x|\sqrt{x^2 + 1}}$

35. $\frac{d}{dx}(\operatorname{sech}^{-1}x) = -\frac{1}{x\sqrt{1 - x^2}}$

36. $\frac{d}{dx}(\coth^{-1}x) = \frac{1}{1 - x^2}$